# Engineering Notes

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## Simple Relations for Analysis of Airbreathing Launch Vehicles

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### Introduction

THE possibility of replacing rocket propulsion by airbreathing propulsion for some phase of the ascent of a launch vehicle has been a recurrent debate for decades.[1,2] In the last few years, this debate has resurfaced with perhaps more vigor than ever before. Despite all of this interest, there still remains the need for some fundamental system studies. Consequently, a set of simple analytical relations is presented in this Note that describes vehicle mass fractions in terms of a velocity parameter ($\alpha$) and a specific impulse parameter ($\beta$). It is hoped that these relations will provide a consistent and objective framework to begin optimization of any airbreathing launch vehicle (ABLV). The relations will be presented for a single-stage-to-orbit (SSTO) vehicle, not because this is necessarily preferred to multiple-stage vehicles, but because the relations then remain succinct. First, some ideal relations and a few inferences are presented, followed by a discussion of a method to transform the relations to include essential drag and gravity losses.

### Ideal Relations

It is simple to write down the burnout mass $m_{br}$, total fuel mass $m_{fr}$, and oxidizer mass $m_{xr}$ of an "ideal" rocket with overall mass $m_o$ (gross liftoff mass) as

$$m_{br}/m_o = \Omega, \tag{1a}$$

$$m_{fr}/m_o = (1-\Omega)/(1+\mu_r) \tag{1b}$$

$$m_{xr}/m_o = (1-\Omega)\mu_r/(1+\mu_r) \tag{1c}$$

where $\mu_r$ is the oxidizer-to-fuel mixture ratio, $\Omega = \exp(-V_{\text{leo}}/I_{sr})$, $V_{\text{leo}}$ is the burnout velocity for low Earth orbit, and $I_{sr}$ is the specific impulse (Ns/kg) assumed constant. In the same manner, it is possible to write down the variations of burnout, fuel, and oxidizer masses for any SSTO ABLV with the same (fixed) overall mass $m_o$. If the vehicle has variable specific impulse $I_s$ over an airbreathing (or combined-cycle[2]) velocity range $V = 0$ to $V = V_\alpha$ and a constant specific impulse $I_{sr}$ from $V_\alpha$ up to $V_{\text{leo}}$, the relations are

$$m_b/m_o = \Omega^{1-\alpha\beta} \tag{2}$$

$$m_f/m_o = [(\Omega^\alpha - \Omega)/(1+\mu_r) + (\Omega^{\alpha\beta} - \Omega^\alpha)/(1+\mu_\alpha)]\,\Omega^{-\alpha\beta} \tag{3}$$

$$m_x/m_o = [(\Omega^\alpha - \Omega)\mu_r/(1+\mu_r) + (\Omega^{\alpha\beta} - \Omega^\alpha)\mu_\alpha/(1+\mu_\alpha)]\,\Omega^{-\alpha\beta} \tag{4}$$

where $\mu_\alpha$ is the vehicle's oxidizer-to-fuel mixture ratio used in the airbreathing phase (often assumed to be zero in ABLV studies), $\alpha$ the fraction of orbital velocity provided by airbreathing propulsion $V_\alpha/V_{\text{leo}}$, and $\beta$ the normalized (average) specific propellant consumption advantage of airbreathing over rocket propulsion, given by

$$\beta = 1 - \frac{I_{sr}}{V_\alpha}\int_0^{V_\alpha}\frac{\mathrm{d}V}{I_s} = (c_{sr} - \langle c_s\rangle)/c_{sr} \tag{5}$$

where $c_{sr} = 1/I_{sr}$ is the rocket's specific propellant consumption, and $\langle c_s\rangle$ is the average value of the specific propellant consumption during the airbreathing phase,

$$\langle c_s\rangle = \frac{1}{V_\alpha}\int_0^{V_\alpha}\frac{\mathrm{d}V}{I_s} \tag{6}$$

Note that, in the case $\alpha = 0$, the relations collapse to the ideal rocket case, and to reduce the propellant mass fraction $1 - m_b/m_o$, $\alpha\beta$ has to be increased in Eq. (2). The reduction in oxidizer mass fraction [Eq. (4)] can, however, be accompanied by an increase in fuel fraction [Eq. (3)]. Equations (2–4) can be differentiated partially with respect to $\alpha$, $\beta$, or $\Omega$. Provided $\mu_\alpha < \mu_r$, it is possible to show the oxidizer mass fraction always falls with $\alpha$, but the fuel mass fraction can either fall or rise depending on whether $\beta$ is greater than a critical value $\beta_{\text{cr}}$,

$$\beta_{\text{cr}} = 1/\left\{1 + \Omega^{1-\alpha}/[(1+\mu_r)/(1+\mu_\alpha) - 1]\right\} \tag{7}$$

If $\beta$ can exceed this critical value, then the fuel mass fraction will fall as $\alpha$ is extended; if not, it will rise.

Hereafter, let us restrict attention to liquid hydrogen and liquid oxygen as the intrinsic vehicle propellants (air being the extrinsic oxidizer) and also assume that $\mu_\alpha = 0$ and $\mu_r = 6$. There now remains only a small region near $\beta = 1$, where the fuel fraction can be smaller than the ideal rocket's. Indeed, it can be demonstrated that even this small region is unlikely to be accessible, because of sensible performance limits. Suppose hydrogen is burned in the airbreathing phase with a mass flow rate $\dot{m}_f$ and heat content $h_f$, about 115 MJ/kg. The propulsive efficiency $\eta_p$ cannot sensibly exceed unity,

$$\eta_p = TV/(\dot{m}_f h_f) = I_s V/h_f < 1 \tag{8}$$

where $T$ is the net thrust, not including separate vehicle drag $D$. Substituting this bound into Eq. (5), integrating and inserting plausible values gives

$$\beta < 1 - V_\alpha I_{sr}/(2\,h_f) \simeq 1 - 0.15\alpha \tag{9}$$

Similarly, $I_s$ cannot exceed some practical multiple $N$ of $I_{sr}$, $I_s < NI_{sr}$, so $\beta < 1 - 1/N$. Hence, if $N = 20$ (a large value), $\beta < 0.95$. Therefore, this ABLV's fuel mass fraction must be

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larger than that of the equivalent ideal rocket, although its propellant mass fraction $(m_f + m_x)/m_o$ will be lower than that of the equivalent ideal rocket provided $\beta > 0$.

Typically, an increase in fuel mass fraction will always accompany a decrease in oxidizer mass fraction as $\alpha$ is extended. Since the density of the fuel $\rho_f$ is, in this case, about 16 times lower than the density of the oxidizer $\rho_x$, the total tankage volume will also typically increase. This growth (at fixed $m_o$) will lead to an increase in structural and thermal protection system masses. To illustrate this, the mass of an integrated tank-wall body structure $m_s$ could be estimated directly from Eqs. (3) and (4) to be

$$m_s/m_o = w_s A/m_o, \qquad A = k(m_x/\rho_x + m_f/\rho_f)^{2/3} \tag{10}$$

where $A$ is the body surface area, $k$ a nondimensional measure of area-volume efficiency (typically about 10), and $w_s$ the specific wall mass. On ascent, ABLV's may enter more severe dynamic pressure[3,4] and thermal environments than ballistic vehicles, essentially to maintain airbreathing engine thrust. Consequently, specific wall mass might rise above likely all-rocket values (20–25 kg/m$^2$); hence, growth in the structural mass fraction $m_s/m_o$ becomes dominant, particularly for vehicles in the lower $m_o$ range (less than, say, 250,000 kg) and/or those accelerating in the airbreathing mode to a high fraction of orbital velocity, i.e., high $\alpha$. The structural mass fraction is not the only term that has an impact on $m_b/m_o$; the engine mass fraction $m_e/m_o$ and contributions from other subsystems/structural elements also must be subtracted before the cargo capacity (payload) $m_c$ can be prescribed.

In simple launch vehicle analysis, it is usual to introduce two parameters to describe mass breakdown; the cargo-to-noncargo mass ratio $\lambda = m_c/(m_o - m_c)$ and the structural efficiency $\sigma = m_d/(m_o - m_c)$, where $m_d = m_b - m_c > m_s$. Combining these parameters with Eq. (2) yields

$$\lambda = (\Omega^{1-\alpha\beta} - \sigma)/(1 - \Omega^{1-\alpha\beta}) \tag{11}$$

This relation can be partially differentiated with respect to structural efficiency (for example), to obtain a measure of cargo mass sensitivity,

$$(\sigma/\lambda)\frac{\partial\lambda}{\partial\sigma} = \sigma/(\sigma - \Omega^{1-\alpha\beta}) < 0 \tag{12}$$

This term has importance because it is often stated that the cargo mass sensitivity of SSTO vehicles implies high "risk." Here, it can be seen that the cargo-to-noncargo mass ratio can be improved, but only if the expected growth in minimum feasible structural efficiency can be suppressed.

## Incorporation of Drag and Gravity Losses

Critics would argue the exact relations presented have little value, because they do not include vehicle drag and gravity losses. One accepted way[5,6] that these losses can be included in the airbreathing phase is by introducing an "effective specific impulse," defined by

$$I_{s^*} = m\left(\frac{dV}{dt}\right)I_s/T = I_s(1 - D/T - W\sin\phi/T) \tag{13}$$

where $W = mg$ is the vehicle flight weight ($m$ is the vehicle flight mass), and $\phi$ is the flight-path angle, with which the vehicle thrust vector is assumed to be parallel. Hence, $\beta$ can be replaced by an effective value

$$\beta \mapsto \beta_* = 1 - \frac{I_{sr}}{V_\alpha}\int_0^{V_\alpha}\frac{dV}{I_{s^*}} \tag{14}$$

In practice, it is only possible to make estimates of $\beta_*$, because detailed assumptions have to made about the vehicle system and its ascent trajectory.[2] However, there is an advantage in that all of these detailed assumptions are contained within a single figure of merit. Equations (13) and (14) show that $\beta_*$ is dependent on airbreathing engine $I_s$, the vehicle thrust-to-weight ratio $T/W$, and, indirectly, the vehicle lift-to-drag ratio $L/D$. Moreover, there is a strong interdependence of these variables with the ascent dynamics, i.e., $V$ and altitude, hence, $\phi(V)$. In general, an increase in $T/W$ improves $\beta_*$, leading to a tradeoff[1,2] of engine mass $m_e$ with $m_b$ $m_x$ $m_f$ and $m_s$. Also, note that the drag $D$ is related to tank size, and self-coupling in the relations becomes evident. Quite simply, if $\beta_*$ falls, tank size increases, $T - D$ decreases, so $\beta_*$ falls still further.[2] The alternative to this "bootstrap" is to prevent $T - D$ falling by increasing $T$, leading to increased engine mass. To the designer, this implies the need for margins to prevent underestimation of performance.

Now that losses in the airbreathing phase have been accounted for, critics might continue to argue that, in the rocket phase of the ABLV ascent, drag and gravity losses also need to be included. A familiar method to deal with this in all-rocket analysis is to introduce a velocity loss term $\Delta V_*$, which increases the total "delta-$V$." The same method can be applied here, so that $V_{\rm leo}$ is replaced by $V_1 = V_{\rm leo} + \Delta V_*$; i.e., the rocket phase delta-$V$ becomes $V_1 - V_\alpha$. The values of $\alpha$ and $\Omega$ are therefore transformed,

$$\alpha \mapsto \alpha_* = V_\alpha/V_1$$

$$\Omega \mapsto \Omega_* = \exp(-V_1/I_{sr})$$

The transformed or "starred" values of $\alpha$, $\beta$, and $\Omega$ now can all be substituted into Eqs. (2–4) without disturbing their form or behavior, although, strictly, it should be noted that $\Delta V_*$ (hence, $\Omega_*$) will be dependent on $\alpha$. Providing that the input values $\alpha_*$, $\beta_*$, $\Omega_*$, $\mu_\alpha$, $\mu_r$, and $I_{sr}$ are all specified, the relations remain exact and objective.

## Numerical Example

Consider a proposal for an SSTO ABLV with overall mass $m_o = 250{,}000$ kg, which has an airbreathing phase up to $V_\alpha = 1490$ m/s (about Mach 5), is launched in an eastward direction near the equator so that $V_1$ is estimated to be 7970 m/s, i.e., $\alpha_* = 0.187$. The vehicle propellants are liquid hydrogen and oxygen. During the airbreathing phase, no liquid oxygen is consumed (or collected); hence, $\mu_\alpha = 0$. In the rocket phase, the rocket engines burn with a mixture ratio of 6 (hence, $\mu_r = 6$) and have a vacuum specific impulse $I_{s\text{-vac}} = 4464$ Ns/kg (455 s). Suppose it is claimed that the vehicle can achieve $\beta_* = 0.55$ with $I_{sr} = I_{s\text{vac}}$; hence, Eqs. (2–4) can be used to find the mass breakdown: $m_b \simeq 50{,}200$ kg, $m_f \simeq 58{,}500$ kg, and $m_x \simeq 141{,}300$ kg. Note also, for example, it can be shown that increasing the maximum airbreathing velocity $V_\alpha$ (at fixed $V_1$ and $\beta_*$) causes the fuel mass to rise since $\beta_* < \beta_{\rm cr} \simeq 0.962$ in Eq. (7).

Now consider a second SSTO ABLV, which has the same $m_o$, $I_{sr}$, $I_{s\text{vac}}$, $\mu_\alpha$, $\mu_r$, and approximately the same $V_1$, but which extends the airbreathing phase (with supersonic combustion between about Mach 5 and Mach 15) to $V_\alpha = 4470$ m/s. If this second vehicle can achieve the same $\beta_*$, its mass breakdown then would be $m_b \simeq 72{,}700$ kg, $m_f \simeq 103{,}000$ kg, and $m_x \simeq 74{,}300$ kg. Note that the burnout mass of the second ABLV is about 45% larger than the first ABLV, but the total ascent propellant volume is about 60% larger. If it is assumed that both vehicles have the same specific wall mass and body area-volume efficiency, then Eq. (10) suggests the second ABLV will have a body structure that is 37% heavier. In view of the fact that the second ABLV must carry a supersonic combustion engine, it is far from clear which vehicle has the larger cargo capacity. Equations (2–4), however, at least provide a framework for tackling this problem.

## Concluding Remarks

Equations (2–4) have been shown to give some useful insights to single-stage-to-orbit (SSTO) airbreathing launch vehicle (ABLV) behavior. They also provide a consistent framework for optimization and comparisons of performance.[1,2]

The relations have been presented for SSTO, but they can be modified for two- (or multiple-) stage vehicles by replacing $V_{leo}$ with the burnout velocity of the airbreathing stage.

Some of the design inferences made here arose because it was assumed the SSTO ABLV consumed only hydrogen in the airbreathing phase. Whereas this choice of fuel may be mandatory for vehicles whose maximum airbreathing velocity extends to large fractions of orbital velocity (in the supersonic combustion regime), it is possible that higher density hydrocarbons are competitive at lower velocities. Dual fuel combinations might offer some relief to the problem of tank volume growth, and relations such as Eqs. (2–4) can be constructed to account for this variation. Similarly, there are interesting tradeoffs to consider with vehicles that use intrinsic oxidizer in the airbreathing phase, i.e., $\mu_\alpha > 0$.

Our problem is to explore and find the best points in the $\alpha$–$\beta$ plane, then to decide whether any of these points represent a real advantage over all-rocket vehicles. There is no doubt that propellant mass fraction can be reduced by airbreathing; however, regarding payload and dry mass fractions, the question is far more complicated[1–4] and beyond the aims of this brief Note.

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# Large Solar Flare Radiation Shielding Requirements for Manned Interplanetary Missions

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## Introduction

As the 21st century approaches, there is an ever-increasing interest in launching manned missions to Mars. A major concern to mission planners is exposure of the flight crews to highly penetrating and damaging space radiations. Beyond the protective covering of the Earth's magnetosphere, the two main sources of these radiations are galactic cosmic rays and solar particle events. Preliminary analyses of potential exposures from galactic cosmic rays (GCR's) were presented elsewhere.[1,2] In this Note, estimates of shielding thicknesses required to protect astronauts on interplanetary missions from the effects of large solar flare events are presented. The calculations use integral proton fluences[3,4] for the February 1956, November 1960, and August 1972 solar particle events as inputs into the NASA Langley Research Center nucleon transport code BRYNTRN.[5] This deterministic computer code transports primary protons and secondary protons and neutrons through any number of layers of target material of arbitrary thickness and composition. Contributions from target nucleus breakup (fragmentation) and recoil are also included. The results for each flare are presented as estimates of dose equivalent [in units of roentgen equivalent man (rem)] to the skin, eye, and bloodforming organs (BFO) behind various thicknesses of aluminum shielding. These results indicate that the February 1956 event was the most penetrating; however, the August 1972 event, the largest ever recorded, could have been mission- or life-threatening for thinly shielded ($\leq 5$ g/cm$^2$) spacecraft. Also presented are estimates of the thicknesses of water shielding required to reduce the BFO dose equivalent to currently recommended astronaut exposure limits.[6] These latter results suggest that organic polymers, similar to water, appear to be a much more desirable shielding material than aluminum.

## Calculational Methods

The incident solar flare integral spectrum[3,4] for each of the three flare events is transported through the shield materials using the Langley Research Center deterministic nucleon transport code BRYNTRN.[5] This code uses a marching algorithm based on integral equation solutions to the one-dimensional Boltzmann transport equation. In the straight-ahead approximation, which is appropriate for these energetic particles, this integrodifferential equation is written as

$$\left[\frac{\partial}{\partial x} - \frac{\partial}{\partial E} S(E) + \sigma_p(E)\right]\phi_p(x,E) = \sum_j \int_E^\infty f_{pj}(E,E')\phi_j(x,E')\,\mathrm{d}E' \qquad (1)$$

for protons. For neutrons, it becomes

$$\left[\frac{\partial}{\partial x} + \sigma_n(E)\right]\phi_n(x,E) = \sum_j \int_E^\infty f_{nj}(E,E')\phi_j(x,E')\,\mathrm{d}E' \qquad (2)$$

where $\phi_j(x,E)$ is the type $j$ particle flux/fluence at $x$ with energy $E$; $S(E)$ is proton stopping power; $\sigma_p(E)$, $\sigma_n(E)$ are proton and neutron total cross sections, respectively; and $f_{ij}(E,E')$ are differential cross sections for elastic and nonelastic processes. Using the detailed solution methods described in Ref. 5, particle fluxes/fluences as a function of depth and energy are computed. In addition to propagating neutrons and protons, contributions from target nuclear fragments and nuclear recoil are also included. The computational algorithm used to calculate these fluences has been verified to within 1% accuracy by comparison with an analytical benchmark solution[7] to Eqs. (1) and (2). Once the particle fluences are known, the energy absorption per gram of material (dose) can be computed from

$$D_j(x, >E) = A_j \int_E^\infty S(E')\phi_j(x,E')\,\mathrm{d}E' \qquad (3)$$